

# Determination of the Frequency Dependent Thévenin Equivalent of Inverters Using Differential Impedance Spectroscopy

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**Abstract**—This paper introduces a measurement technique that allows grid-connected inverters to be interpreted as frequency-dependent Thévenin equivalents. By means of this so-called differential impedance spectroscopy it is possible to determine the spectrum of internal harmonic voltage sources as well as the effective frequency-dependent output impedance of an inverter. Having this, one can distinguish between resonance-based and source-driven harmonics, which provides a more useful analysis of harmonic interactions between inverters and the grid. This paper presents the fundamentals and the procedure of the differential impedance spectroscopy, describes a realized test-stand for inverters up to one megawatt and gives exemplary measurement results.

**Index Terms**—Impedance spectroscopy, impedance based stability, stability analysis, harmonic sources, output impedance of inverters, grid stability

## I. INTRODUCTION

Together with the rising share of renewable energy sources in final energy consumption and the further integration of other power electronics based systems such as battery storages, electrolyzers, HVDC transmission systems, charging stations for electric vehicles, etc. new challenges for stable grid operation arise. Among others, resonances between

cables and power electronic devices, interactions of controllers and passive components, and new power oscillations ranked among the top challenges seen by European transmission system operators when it comes to large penetration of power electronic based generation [1]. A major technical challenge is to ensure stable grid operation and maintain a high voltage quality. However, problems with critical harmonic oscillations in PV and wind farms can already be observed, which are due to electrical resonance between inverters and the grid. With established methods for determining harmonic emissions, almost no statements can be made about possible resonances and associated consequences. One promising approach to analyze resonance-based oscillation is the impedance-based stability criterion [2]. For its use the effective impedance of given inverter and the grid connection point must be known. However, measuring the effective output impedance of inverters is not facile due to the presence of internal harmonic sources, which may disturb the measuring results as discussed in the following section. In order to overcome this problem the inverter can be considered as a set of frequency-depending Thévenin equivalents. By determining both the impedance and the internal voltage sources of the Thévenin equivalents one can distinguish between resonance-based and source-

The presented work was financially supported by the German Federal Ministry for Economic Affairs and Energy under the project number 0325757F („Netzharmonie“).

978-1-7281-6990-3/20/\$31.00 ©2020 European Union

driven harmonics [3]. Sections III and IV present the *differential impedance spectroscopy*, which is a measurement technique that allows to determine the representing Thévenin equivalents of inverters. In Section III and IV a measurement technique is presented which makes it possible to determine the representing Thévenin equivalents of inverters. This approach makes it possible to explain the influence of the grid impedance on the harmonic emissions of an inverter, which established methods cannot do due to their limitations. Furthermore, the superposition of harmonic emission from multiple inverters can be improved by having a Thévenin equation model of a given inverter. Section V and VI show exemplary results for different solar inverters and demonstrate how resonance problems between inverters and the power grid can be investigated by means of the impedance-based stability criterion.

## II. IMPEDANCE SPECTROSCOPY OF INVERTERS

Impedance spectroscopy is generally understood to be the metrological determination of the complex impedance curve of the examined system by means of a frequency-variable voltage excitation. Impedance spectroscopy has a long history and is a commonly used method to analyze dielectric materials [4]. So far it is assumed that the systems to be characterized by impedance spectroscopy consist of a network of linear passive components. When analyzing inverters, the presence of internal harmonic sources must be taken into account, as shown below. Another difference is that a supply voltage with the fundamental frequency  $f_{fund}$  (50 Hz or 60 Hz) must be provided first in order to set the device under test (DUT) into operation. An excitation voltage  $V_{exc}$  with the frequency  $f_{exc}$  must then be superimposed on that. In figure 1 the setup of an impedance spectroscopy measurement of an inverter is depicted. The voltages and currents at the terminals of the DUT are measured and transformed into the frequency domain. However, only the excitation frequency component is considered for further data processing. This gives the measured values for the voltage excitation  $\underline{V}(f_{exc})$  and the corresponding current response of the inverter  $\underline{I}(f_{exc})$ . Subsequently, the excitation frequency is increased in practical steps over the frequency range under consideration.

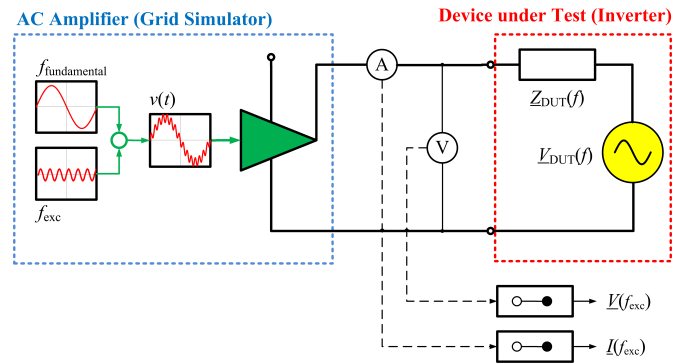


Figure 1: Principle of the impedance spectroscopy of inverters (single line representation).

A test stand comprising a 1 MVA high-dynamic AC amplifier according to the setup described above was established at Fraunhofer ISE's *Multi-Megawatt Lab*. By calculating the ratio between  $\underline{V}(f_{exc})$  and  $\underline{I}(f_{exc})$  for each analyzed frequency one can get a kind of impedance characteristic of the inverter. Figure 2 shows the calculated ratio for a 20 kVA solar inverter in an exemplary manner.

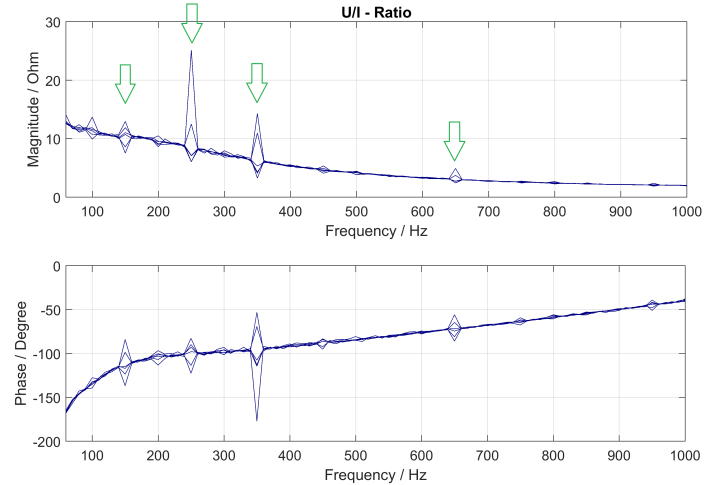


Figure 2: Exemplary frequency-depending voltage to current ratio ( $V/I$  ratio) of a 20 kVA solar inverter.

However, we notice, that the curve shows discontinuities at different frequencies (cf. markers at 150 Hz, 250 Hz, 350 Hz and 650 Hz). Since these discontinuities cannot be traced back to the behavior of passive electrical components, they are supposed to be caused by the presence of internal harmonic sources at exactly these frequencies. Hence, instead of calling this ratio 'impedance' it should be titled

as *frequency-dependent voltage to current ratio (V/I ratio)* [5]. In order to determine these internal harmonic sources and to get rid of the discontinuities in the impedance curve, a more sophisticated approach is required, which will be explained in the following section.

### III. DETERMINATION OF THÉVENIN EQUIVALENTS BY MEANS OF DIFFERENTIAL IMPEDANCE SPECTROSCOPY

The Thévenin theorem states that any combination of impedances and voltage or current sources can be represented by a voltage source behind an equivalent impedance [6]. This so-called Thévenin equivalent describes fully the small-signal behavior of the represented two-pole at its terminals. Provided that an inverter can be considered as Thévenin equivalents in the frequency domain for a given operational point, the task consists in determining the impedance  $\underline{Z}_{DUT}(f)$  as well as the voltage source  $\underline{V}_{DUT}(f)$  frequency-wise. In order to do so, one measurement with one excitation per frequency is not sufficient to specify the two unknowns. To solve this problem at least two independent measurements with different excitations are required. Therefore, the DUT will successively be stimulated by excitation voltages which differ in phase and/or amplitude. Having the measurement results of two independent measurements at one's disposal one can set up the equations system in (1), where  $\underline{V}_A(f)$  and  $\underline{I}_A(f)$  represent the measured voltage and current of the first excitation and  $\underline{V}_B(f)$  and  $\underline{I}_B(f)$  the measurement values of the second excitation.

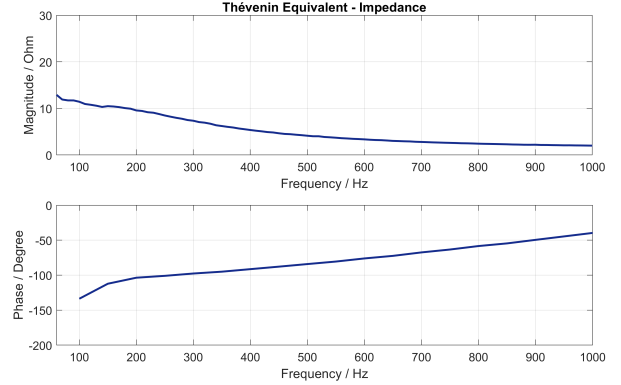
$$\begin{aligned} \underline{V}_A(f) &= \underline{I}_A(f) \cdot \underline{Z}_{DUT}(f) + \underline{V}_{DUT}(f) \\ \underline{V}_B(f) &= \underline{I}_B(f) \cdot \underline{Z}_{DUT}(f) + \underline{V}_{DUT}(f) \end{aligned} \quad (1)$$

Solving this simple equation system for  $\underline{Z}_{DUT}(f)$  and for  $\underline{V}_{DUT}(f)$  leads to the following equations for the elements of the Thévenin equivalent (2):

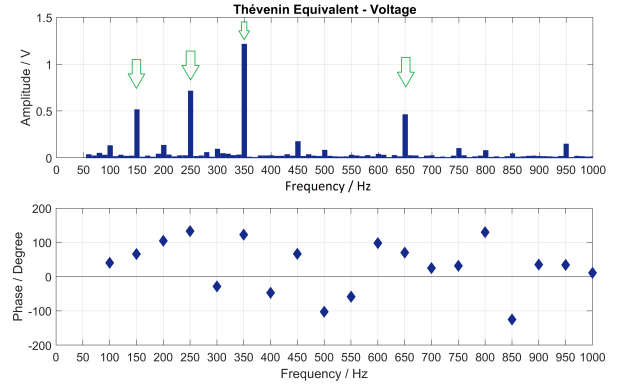
$$\begin{aligned} \underline{Z}_{DUT}(f) &= \frac{\underline{V}_B(f) - \underline{V}_A(f)}{\underline{I}_B(f) - \underline{I}_A(f)} \\ \underline{V}_{DUT}(f) &= \frac{\underline{V}_A(f) \cdot \underline{I}_B(f) - \underline{V}_B(f) \cdot \underline{I}_A(f)}{\underline{I}_B(f) - \underline{I}_A(f)} \end{aligned} \quad (2)$$

Since  $\underline{Z}_{DUT}(f)$  symbols the frequency-dependent differential impedance of the DUT, this method is

called *differential impedance spectroscopy*. Figure 3a shows the impedance of the Thévenin equivalent, which was determined by means of differential impedance spectroscopy for the same 20 kV A solar inverter as before. By comparing figure 3a with figure 2 it becomes obvious that the discontinuities are completely removed in the Thévenin impedance.



(a) Frequency-depending impedance of the Thévenin equivalents.



(b) Voltage spectrum of the Thévenin equivalents.

Figure 3: Exemplary results for a differential impedance spectroscopy measurement of a 20 kVA solar inverter.

Furthermore, figure 3b depicts the identified voltage spectrum of the Thévenin equivalents. It turns out that significant voltage sources appear at those frequencies that showed discontinuities in the V/I ratio (cf. 150 Hz, 250 Hz, 350 Hz and 650 Hz) before. For validation purposes and for better accuracy more than two different excitations per frequency step can be applied. Consequently this leads to an over-determined equation system and allows for a best-fit solution and a deviation factor for the determined values of the Thévenin equivalent.

#### IV. BUILT-UP TEST BENCH

Figure 4 shows the built-up test bench for the impedance spectroscopy explained above. The device under test (DUT) is first supplied on the input side from a DC source.

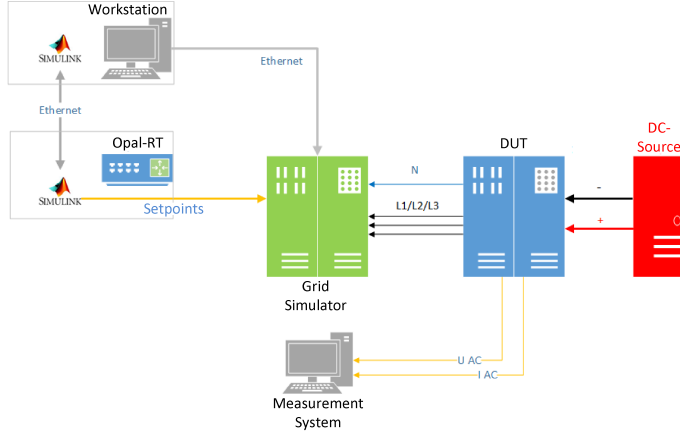


Figure 4: Built-up test bench for the impedance spectroscopy of inverters

For testing a solar inverter, it is advisable to use a DC source with a configurable output characteristic, which enables the emulation of the I-V curve of a PV array. The core component of the test bench is a highly dynamic AC amplifier, which generates the AC voltage for the device under test. The voltage setpoints are transmitted via an Opal RT real-time system to the power amplifier of the grid simulator. A MATLAB/Simulink based program can be used for parametrization and monitoring issues, especially to set the harmonic excitation. Furthermore, the general configuration and operation of the grid simulator is done via a workstation computer. The current and voltage measured values at the terminals of the device under test are recorded by a power meter for later evaluation. To be able to record frequencies up to 10 kHz correctly, a sampling frequency of more than 25 kHz should be selected. The measurements for this work were all recorded with a sampling frequency of 50 kHz. The highly dynamic power amplifier has a total power of 1 MV A and a bandwidth of 10 kHz. Since the simulator has to supply the DUT, devices up to 1 MV A can be tested within their full power range. The AC amplifier allows to supply devices up to an nominal AC-Voltage of 1000 V. Being a real technical system, the AC amplifier is not an ideal voltage source and

shows some low output impedance. Anyhow, this circumstance does not influence the results of the impedance spectroscopy, because the actual measured voltage at the terminals of the DUT is considered for the analysis. Nevertheless, resonances between the output impedances of the grid simulator and the DUT must be avoided in order to keep the voltage excitation in the small-signal range.

#### V. COMPARISON OF DIFFERENT INVERTERS

In this chapter the Thévenin impedances of different solar inverters are compared. All devices are three-phase, solar inverters in the power range of 10 kW to 2.5 MW. Beside the power rating the DUTs differ in their bridge and filter topologies. Table I shows the four inverter being compared with each other, their nominal powers and the power during the spectroscopy. It was demonstrated that the operating point of the inverter influences the course of the impedance. The influence is strongly device-dependent because it is clearly visible at DUT 3 and negligible at DUT 4. We mention this only for the sake of completeness, but will not discuss it further.

Table I: Devices under test and their nominal power

Device under test	Nominal power	Power at operation
DUT 1	23 kW	23 kW
DUT 2	1 MW	500 kW
DUT 3	150 kW	150 kW
DUT 4	2.5 MW	500 kW

In Figure 5 the Thévenin impedances for the four inverters are depicted. One can see that the curves differ greatly from each other in form and magnitude. The two devices in the lower power class have an impedance in the range from  $1 \Omega$  to  $10 \Omega$ . The devices in the megawatt class have an impedance in the range of  $0.02 \Omega$  to  $10 \Omega$ . The course of the curves in the upper frequency range is mainly determined by the passive output filter components. Depending on used filter topology, one or more resonance points can be found. In the low frequency range, where the inverter's current controller has an influence on the electrical behavior, the course of the impedance is strongly determined by the inverter control. Figure 6 shows the phases of the Thévenin impedance. As to the amounts of impedance large differences are visible here.

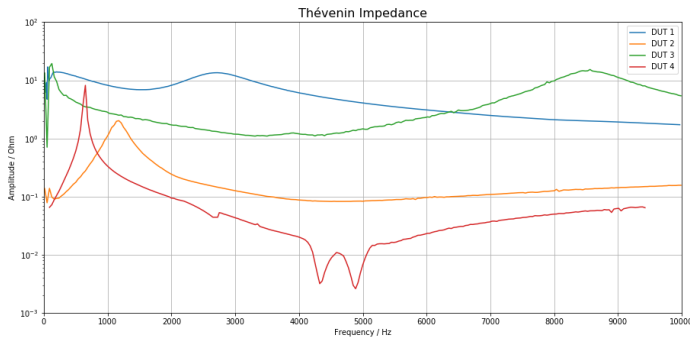


Figure 5: Magnitude of the Thévenin impedance for different solar inverters

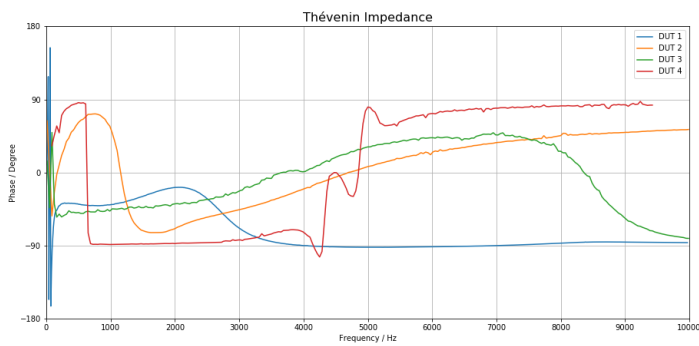


Figure 6: Phase of the Thévenin impedance for different solar inverters

## VI. MEASUREMENT RESULTS AND APPLICATION OF THE IMPEDANCE-BASED STABILITY CRITERION

In this chapter exemplary results of differential impedance spectroscopy measurements for different solar inverters with an apparent power up to 1 MVA are presented and compared with each other. The suitability of impedance characterization of inverters can be demonstrated by the following experiment. Given was a 30 kVA solar inverter, which showed significant harmonic oscillation at 5 kHz, when being operated at a given grid impedance. The oscillations can be seen in the spectrum as well as in the instantaneous values of the inverter current (cf. lower left diagram in figure 7). The impedance-based stability criterion states that resonances may occur when the curves of the grid and inverter impedance magnitude intersect while having a phase shift close to  $180^\circ$ . The analysis of

the impedance characteristic of the grid connection point and the given inverter, which was derived by differential impedance spectroscopy, actually reveals an attenuated resonance point at 5 kHz with a low phase margin of approx.  $20^\circ$  (cf. upper left diagram in figure 7). Another point of intersecting magnitude curves at 1 kHz has poor phase margin, too, which results in higher harmonic content at this frequency, whereas the intersection point at 4.5 kHz shows sufficient phase margin and is uncritical in terms of harmonics. In a second step the inverter control was improved by optimizing the timing of the digital current controller. This led to a better dynamic behavior of the inverter, which has a major impact on the impedance characteristic of the inverter. By comparing the impedance characteristics of the inverter with original and adjusted controller configuration in Figure 7 it is clearly visible, that the resonance point of the LC output filter at 4.8 kHz is strongly attenuated by the optimized controller. As a result the intersection of the magnitude curves of the inverter and the grid impedance disappears. Thus, the resonance at point at 5 kHz disappears, too (cf. diagrams in figure 7 on the right side). This shows that the presented method is suitable to investigate resonance phenomena and evaluate possible counter measures.

## VII. CONCLUSION

The differential impedance spectroscopy represents a test method, which enables both, the determination of the active harmonic emissions of an inverter and the analysis of resonance tendencies at known grid connection points. Therefore an inverter is considered as frequency-dependent Thévenin equivalent, which is determined by measuring the output impedance of inverter as well as the spectrum of internal harmonic voltage sources. In contrast to established methods for analyzing harmonic emissions of the inverters the differential impedance spectroscopy makes it possible to characterize the impact of variation in the grid impedance and grid voltage distortions on the harmonic emissions of inverters. The comparison of different solar inverters has shown that the output impedance strongly depends on the topology and the controller structure. Furthermore, the evaluation of resonance problems can now be investigated by means of the impedance-based



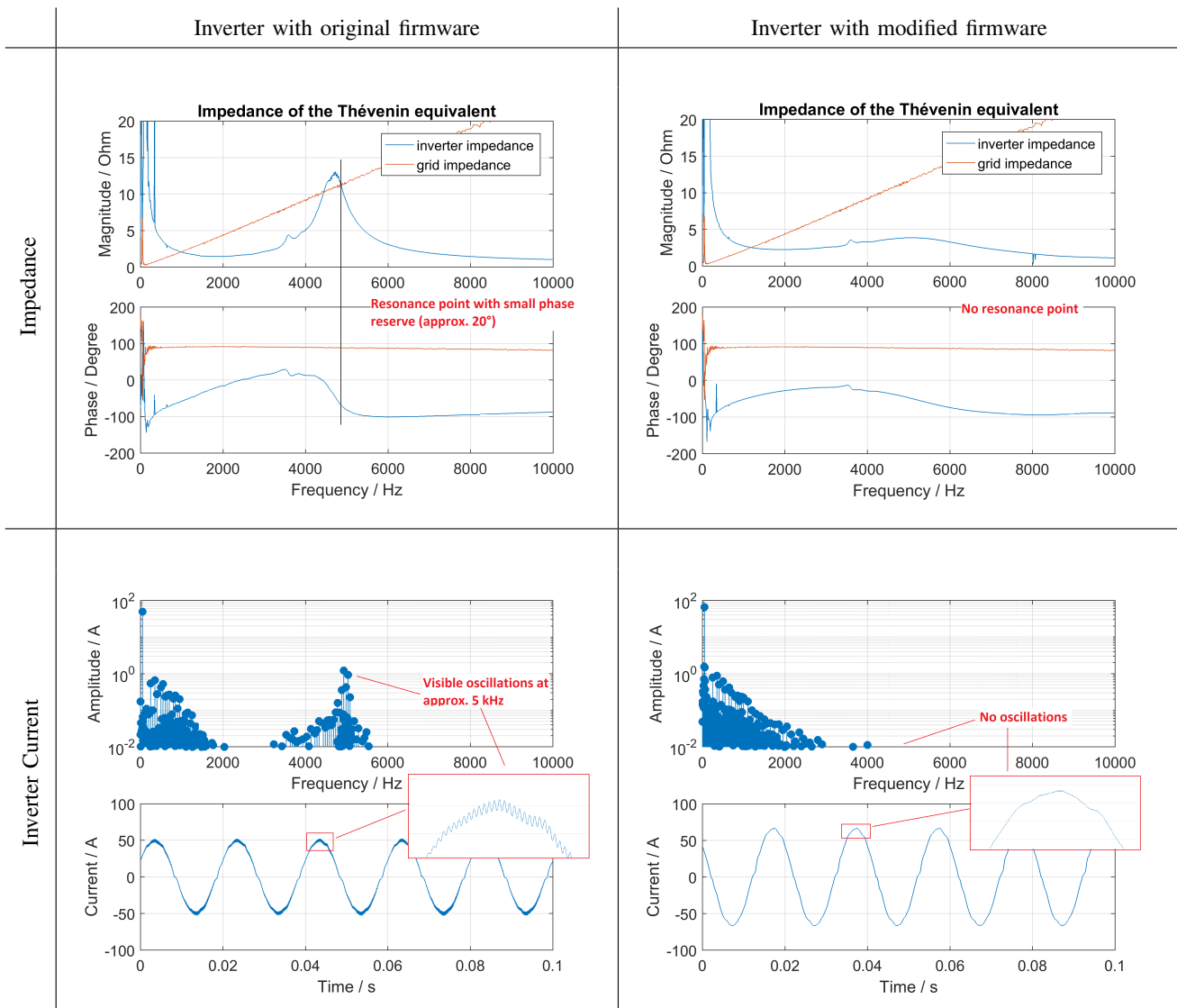


Figure 7: Application of the impedance-based stability criterion for a grid-connected inverter showing a 5 kHz resonance, which disappears when adjusting the controller configuration of the inverter.

stability criterion. The suitability of the method was proven by testing different solar inverters in a lab environment. This results encourages to use of the Thévenin equivalent representation for frequency-domain analysis of harmonic issues in field applications, which will be an interesting task for following research projects.

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